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On the use of Variational Autoencoders for Nonlinear Modal Analysis

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1 Introduction

Linear modal analysis offers a vital and mostly complete framework for the dynamic analysis of simplified engineering systems, with important insights offered from the extraction of natural frequencies and mode shapes. The extracted mode shapes further serve as an invariant basis upon which to build reduced-order models (ROMs) of linear systems [1]. When moving to nonlinear systems, however, the principles upon which modal analysis are based, no longer hold. This issue motivates the development of a framework for nonlinear modal analysis, which can maintain some of the key features of modal analysis [2]. This work is based upon the concept of nonlinear normal modes (NNMs); these consist of invariant manifolds upon which motion for a given NNM is constrained. NNMs can also provide insight into engineering systems as well as offer a basis for construction of nonlinear ROMs [2].

The extraction of these NNMs can be done analytically or numerically; however, some recent work has focussed on the use of machine learning methods for the extraction of NNMs from output-only data. This methodology, initially shown in [3, 4], used statistical independence of the extracted quantities as justification for calling these objects NNMs. The statistical independence is thought to be analogous to the modal invariance exhibited by NNMs. More recent work has specifically demonstrated the power of neural networks, specifically generative adversarial networks, to this problem [5]. Herein we make use of the variational autoencoder (VAE), as another potential neural network-based method for the extraction of NNMs.

The VAE, as first described in [6], is a Bayesian implementation of an autoencoder network, which makes use of variational inference to allow efficient training for large models and data; the architecture of a VAE is illustrated in Figure 1. Being an autoencoder, the purpose of the VAE is to learn a latent space representation of data, along with the transforms to and from this latent space. VAEs are usually used to facilitate dimensionality reduction or de-noising and has indeed often been used for extracting low-dimensional manifolds. The VAE attempts to minimise the evidence lower bound (ELBO) cost function [6]; this cost function is a combination of the error of reconstruction of the autoencoder and the Kullback-Leibler divergence between the latent space variables and a circular Gaussian distribution. By encouraging the latent space to represent a circular Gaussian, this encourages the autoencoder to extract independent latent variables from the data. If the VAE can extract statistically-independent latent variables from the displacement time series of a nonlinear system, it can be considered to be extracting NNM-like quantities; if it can recombine these to reconstruct the response in physical space, then it is also acomplishing a manner of "nonlinear superposition" of these NNM-like quantities .

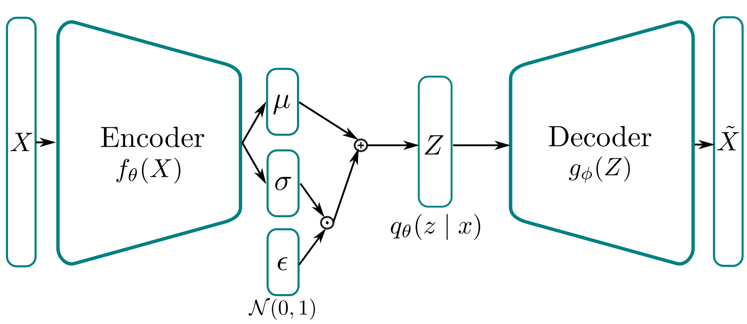

Fig. 1: Architecture of a variational autoencoder (VAE).

2 Application Case Study

To demonstrate the use of VAE's for NNM extraction, a nonlinear experimental dataset was chosen, namely, a three-storey shear-frame structure from Los Alamos National Laboratory (LANL) [7]. Within the three-storey structure, nonlinearity is introduced via a bumper which contacts a column placed between the second and third floors. The bumper introduces a bilinear stiffness-type nonlinearity. For this work, the acceleration at each of the 3 floors is taken as output with the base excitation considered as input. The experiment is carried out in various formulations, in this case, experiment 12 was chosen, with 50 repeats of the experiment, each carried out under band-limited random base excitation.

3 Results and Discussion

As in previous works [3–5], the algorithm will be considered to have successfully extracted uncoupled NNMs if the spectra of the extracted latent state variables contain a single peak. The performance of the VAE is compared against a standard principal component analysis (PCA) [8], decomposition of the system, which is equivalent to linear modal analysis [9]. Figure 2 shows the power spectral density (PSD) of the variables extracted by a PCA decomposition of the experimental data. It is clear that the algorithm cannot successfully decompose the system, because of the significant presence of second and third modal peaks in the resulting PSDs. The failure to decompose the system is a result of the nonlinearities present.

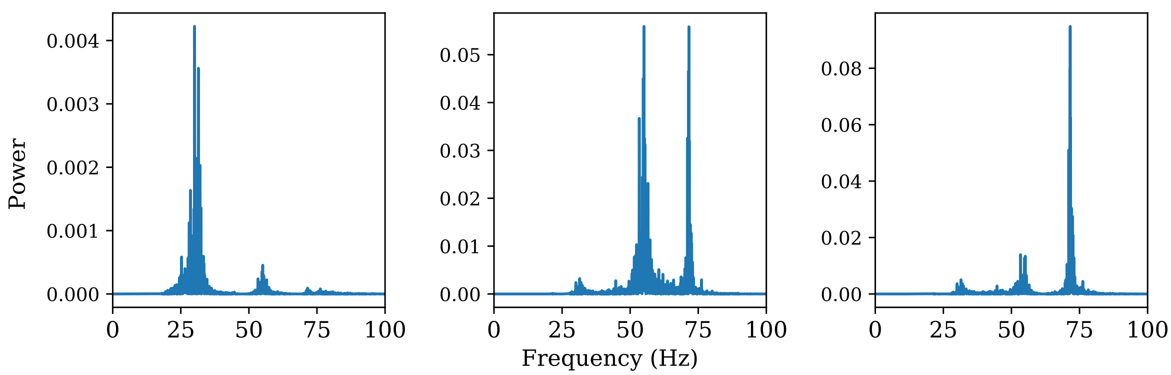

Fig. 2: PSDs of three-floor experimental structure, PCA decomposition.

Figure 3 shows the PSD of the latent variables extracted by the VAE decomposition of the experimental data. It is clear that this decomposition is much more successful, in comparison to PCA, as the extracted components largely contain only a single resonance peak.

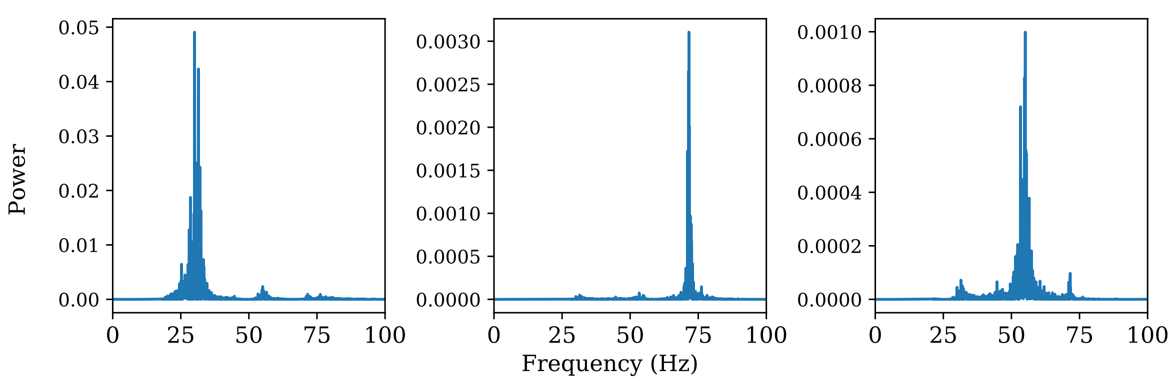

Fig. 3: PSDs of three-floor experimental structure, VAE decomposition.

Finally, for the potential utility of such decompositions for the purpose of reduced-order modelling, it is essential that the inverse transformation, which forms a return mapping from the latent space to physical space, is also successful. In this case, this involves comparing the output of the VAE to the input, to track the quality of the reconstructed response signal. Figure 4 shows the original response data in blue and the data after passing through the VAE in orange. The fidelity of the reconstruction is shown to be of high quality, indicating that the problem of "nonlinear superposition" was also met by the VAE.

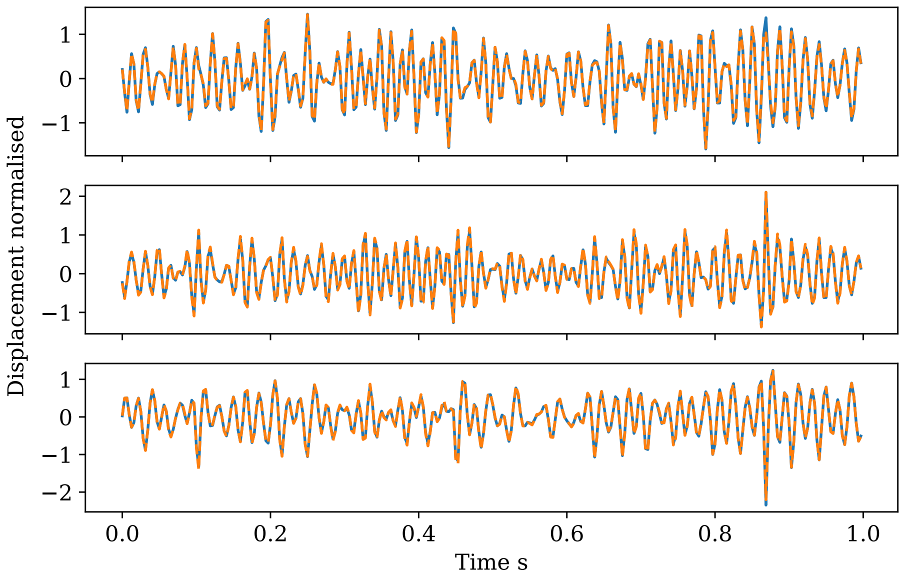

Fig. 4: Superposition/inverse transformation for the experimental system (orange) and original displacements (blue)

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